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The critical laser intensity of self-guided light filaments in air

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Abstract. The critical intensity inside plasma filaments generated in air by high-power, ultra-short laser pulses is estimated analytically and compared to recent experimental data. The result, $I_{\mathrm{crit}} \approx 4\times10^{13}\,\mathrm{W/cm^2}$, is highly relevant for atmospheric applications.

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When propagating in air, high-power femtosecond laser pulses produce self-guided high-intensity plasma filaments, [1–4] the diameter of which could be as small as 100 μm. These filaments are due to a balance between Kerr self-focusing, and defocusing due to the plasma generated inside the filament. Due to the small value of the nonlinear refractive index of air ($n_2 = 3\times10^{19}\,\mathrm{cm^2/W}$), the equilibrium is reached for low plasma densities [5]. A precise description of the filamentation process would need more experimental data relating to parameters such as the intensity inside the filament. Such data would also be useful for atmospheric applications such as lightning control [6] or Lidar [7, 8].

However, due to the very high intensities inside in the filaments (at least $10^{13}\,\mathrm{W/cm^2}$), direct measurements are not possible because any device used for that purpose, whether a detector, a reflector or an attenuator, would be damaged. Therefore, only indirect measurements may be performed.

In this short note, we provide an estimation of the intensity inside a filament generated in air by a femtosecond laser pulse, based on recent experimental data obtained by Talebpour et al. [9]. It is well known [10] that the pulse self-focuses until photoionization sets in. Although the detailed propagation dynamics of the pulse may be quite complicated, its propagation and filamentation is dictated by an equilibrium between the focusing Kerr effect and defocusing multiphoton/tunnel ionization. (In the following, the word ionization will be used alone whenever possible for simplicity). It may be shown that diffraction may, to the first approximation, be neglected. We also neglect all non-instantaneous contributions to nonlinearity, and we shall average all relevant quantities over the whole pulse. Consequently, we obtain a filament whose size and intensity are determined by the following equation, which is a balance between Kerr self-focusing $\Delta n_{\mathrm{Kerr}}(\mathit{neutral})$ and defocusing $\Delta n(\mathit{plasma})$ contributions to the nonlinear index of refraction [1]:

$$\Delta n_{\mathrm{Kerr}}(\mathit{neutral}) \approx \Delta n(\mathit{plasma}) \tag{1}$$

Both changes in the indices of refraction depend on the laser intensity I:

$$n_2 \times I \approx N_{\mathrm{e}}(I)/2N_{\mathrm{crit}} \tag{2}$$

In this expression, n_2 is the Kerr nonlinear index of refraction of the propagation medium, $N_{\mathrm{crit}} = \varepsilon_0 \times m \times \omega^2/e^2$ is the critical plasma density, with m being the mass of electron, ω the laser angular frequency, and e the elementary charge. $N_{\mathrm{crit}} = 1.7\times10^{21}\,\mathrm{cm^{-3}}$ for a titanium–sapphire laser with a central wavelength of 800 nm. The electron density N_{e} is given by ionization:

$$\mathrm{d}N_{\mathrm{e}}(z,t)/\mathrm{d}t = R(I) \times N(z) \tag{3}$$

where $N(z)$ is the density of neutrals as a function of the propagation distance z, and $R(I)$ is the ionization rate. The results of Talepbour et al. [9] exhibit an effective power law dependence of ionization as a function of intensity. This leads us to write R as:

$$R(I) = R_{\mathrm{T}} \times (I/I_{\mathrm{T}})^{\alpha} \tag{4}$$

where R_{T} and I_{T} are a pair of experimental values used as a reference point. They can be estimated for nitrogen and oxygen from the ionization data shown in Fig. 1, adapted from [9]. We chose $I_{\mathrm{T,O_2}} = I_{\mathrm{T,N_2}} = I_{\mathrm{T}} = 10^{13}\,\mathrm{W/cm^2}$. The corresponding ionization rates are $R_{\mathrm{T,N_2}} = 2.5\times10^{4}\,\mathrm{s^{-1}}$ and $R_{\mathrm{T,O_2}} = 2.8\times10^{6}\,\mathrm{s^{-1}}$, respectively. A linear fit of the curves gives the slopes $\alpha_{\mathrm{N_2}} \approx 7.5$ for nitrogen and $\alpha_{\mathrm{O_2}} \approx 6.5$ for oxygen for intensities $I < 10^{14}\,\mathrm{W/cm^2}$. These values are lower than those expected from the number of photons needed for multiphoton ionization, which are 11 and 8 respectively for nitrogen and oxygen at a wavelength of 800 nm, which is

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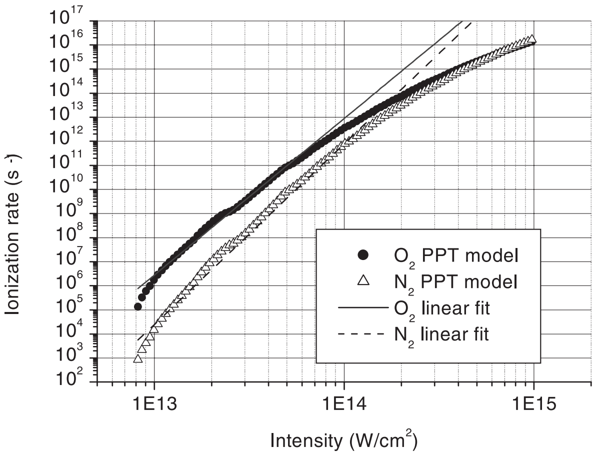

Fig. 1. Tunnel ionization rate of N_2 and O_2 molecules as a function of laser intensity, as calculated using the PPT model by Talepbour et al. [9]

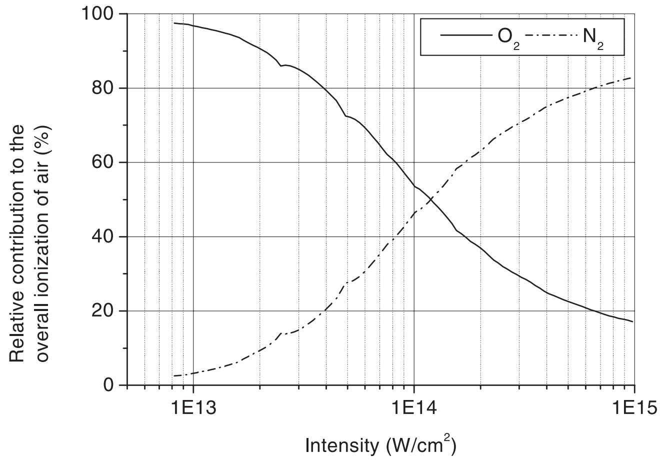

Fig. 2. Relative ion densities from O_2 and N_2 with respect to the total number of ions in a laser-generated plasma in air, as a function of the laser intensity after data from [9]. The small dips in the curves are due to dynamic resonances in the ionization process [18]

an indication of the occurrence of tunnel ionization [11]. We should point out that the ionization rates of oxygen are not much higher than those of nitrogen, even though the ionization potential of oxygen (12.1 eV) is lower than that of nitrogen (15.6 eV) [9]. This is explained by Muth-Böhm et al. [12] as being due to the nature of electron orbitals in the two molecules. In oxygen, with an anti-bonding orbital, electron waves ionized from the two nuclear sites interfere destructively, leading to a significant reduction (suppression) of the ionization probability. In nitrogen, with a bonding orbital, electron waves ionized from the two nuclear sites interfere constructively. Consequently, the probability of ionization is high. As such, in our estimation, we have to keep the contributions to ionization from both O_2 and N_2, and cannot totally neglect the contribution of N_2.

As long as the pulse duration remains shorter than the typical collision time in air, which is about 1 ps in standard conditions, only tunnel/multiphoton ionization occurs, and collisional ionization may be neglected. The treatment of O_2 and N_2 can therefore be decoupled. Equation (3) thus becomes:

$$\begin{aligned} \mathrm{d}N_\mathrm{e}(z,t)/\mathrm{d}t &= \mathrm{d}N_{\mathrm{e,N_2}}(z,t)/\mathrm{d}t + \mathrm{d}N_{\mathrm{e,O_2}}(z,t)/\mathrm{d}t \\ \mathrm{d}N_\mathrm{e}(z,t)/\mathrm{d}t &= N_{\mathrm{N_2}}(z) \times R_{\mathrm{T,N_2}} \times (I/I_\mathrm{T})^{\alpha_{\mathrm{N_2}}} \\ &\quad + N_{\mathrm{O_2}}(z) \times R_{\mathrm{T,O_2}} \times (I/I_\mathrm{T})^{\alpha_{\mathrm{O_2}}} \end{aligned} \quad (5)$$

Here, $N_{\mathrm{N_2}}(z) = 0.78 \times N_{\mathrm{air}}(z)$ and $N_{\mathrm{O_2}}(z) = 0.21 \times N_{\mathrm{air}}(z)$ are the number densities of N_2 and O_2 molecules in air, respectively. At atmospheric pressure, this corresponds to $N_{\mathrm{N_2}}(z) = 2 \times 10^{19}\,\mathrm{cm}^{-3}$ and $N_{\mathrm{O_2}}(z) = 5 \times 10^{18}\,\mathrm{cm}^{-3}$, respectively. The relative contribution of both terms, i.e. $(\mathrm{d}N_{\mathrm{e,N_2}}(z,t)/\mathrm{d}t)/(\mathrm{d}N_\mathrm{e}(z,t)/\mathrm{d}t)$ and $(\mathrm{d}N_{\mathrm{e,O_2}}(z,t)/\mathrm{d}t)/(\mathrm{d}N_\mathrm{e}(z,t)/\mathrm{d}t)$ as a function of the incident laser intensity using (5) is shown in Fig. 2. Both terms have the same order of magnitude in the intensity range of interest, and their relative contribution to ionization cross each other around $10^{14}\,\mathrm{W/cm^2}$. This means that in the general case the contributions of both O_2 and N_2 have to be taken into account. The electron density is therefore estimated from (5):

$$N_\mathrm{e} = \left[R_{\mathrm{N_2}}(I) \times N_{\mathrm{N_2}}(z) + R_{\mathrm{O_2}}(I) \times N_{\mathrm{O_2}}(z)\right] \times \tau_\mathrm{I} \quad (6)$$

where τ_I is a characteristic ionization time which is on the order of the laser pulse duration. If we neglect the z-dependence of N and consider it as a constant, the lowest intensity, or critical intensity I_{crit}, needed to maintain a filament is the solution of (2):

$$n_2 \times I_{\mathrm{crit}} = \frac{\tau_\mathrm{I}}{2 \times N_{\mathrm{crit}}} \left(N_{\mathrm{N_2}} \times R_{\mathrm{T,N_2}} \times (I_{\mathrm{crit}}/I_\mathrm{T})^{\alpha_{\mathrm{N_2}}} + N_{\mathrm{O_2}} \times R_{\mathrm{T,O_2}} \times (I_{\mathrm{crit}}/I_\mathrm{T})^{\alpha_{\mathrm{O_2}}}\right) \quad (7)$$

This equation cannot be solved analytically. For our parameters, with $\tau_\mathrm{I} = 100$ fs, a numerical solution taking into account both N_2 and O_2 gives $I_{\mathrm{crit}} = 4 \times 10^{13}\,\mathrm{W/cm^2}$. At this intensity, Fig. 2 shows that oxygen accounts for more than 80% of the overall plasma. This value is in good agreement with the value of $I = 4.5 \times 10^{13}\,\mathrm{W/cm^2}$ estimated by Lange et al. [13] from the higher harmonics pattern.

Alternatively, we can estimate the intensity in the filament by the following observation. The electron density measured by several groups [5, 14–17] has a wide range of values, ranging from $10^{12}\,\mathrm{cm}^{-3}$ [14] to $3 \times 10^{16}\,\mathrm{cm}^{-3}$ [15]. However, these discrepancies can be explained by the different experimental conditions. In the case of Schillinger et al. [14], the density was averaged over a section with a diameter of 11 mm, containing $10-20$ filaments. Taking into account the fact that the plasma is only to be found in the filament, the equivalent plasma density inside the filaments is in the order of $5 \times 10^{14}\,\mathrm{cm}^{-3}$. The results of Tzortzakis et al. [15] could have been overestimated because they have measured relatively near the focal point of their focusing lens. Therefore, a reasonable range for the free electron density produced by high power ultra-short laser pulses in air is about 10^{14}–$10^{15}\,\mathrm{cm}^{-3}$, corresponding to probabilities of ionization of the order of 10^{-5}–10^{-4}. We can reasonably assume that the focal volume is approximately constant in the filament, and that saturation of the ionization in Figs. 1 and 2 of [9] corresponds to a full ionization of the gases. Therefore an ionization probability of 10^{-5}–10^{-4} corresponds to an intensity of the order of 4–$6 \times 10^{13}\,\mathrm{W/cm^2}$ for oxygen as well as for nitrogen. This is in good agreement with the critical intensity calculated above. Due to the high α value for both N_2 and O_2, even considerable differences in the measured plasma density will not affect the inferred critical intensity substantially.

In conclusion, we have estimated the critical laser intensity in the filaments of intense Ti:Saphire laser pulses propagating in the atmosphere by two independent methods. Both methods agree with each other and give an intensity in the order of $4 \times 10^{13}\,\mathrm{W/cm^2}$. This result is highly relevant for the study of the propagation of high-power femtosecond laser pulses in air, from a fundamental point of view as well for atmospheric applications such as laser lightning and Lidar.

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